PINNeAPPle

STUDY REPORT · PUBLIC EXAMPLE

Surface-PM machine torque computed by two routes

The same torque by Maxwell stress and by co-energy — and the rotor topology optimization that only counts when the two agree.

0.09%

between Maxwell and co-energy at the design point

16.70 Nm

of torque at γ = 0° in the 4-pole machine

$4.6 \cdot 10^{-8}$

maximum error in the gradient check

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Demonstration case: dimensions, materials and coefficients illustrative
2-D A_z FEM · two independent torque routes
PINNeAPPle · LAIS & YAN LTDA · pinneapple-labs.github.io

A surface-mounted permanent-magnet machine in 2-D: cogging, torque against current angle, back-EMF and losses. Torque is computed twice, by independent routes, and both are printed side by side.

QUANTITY	VALUE
Poles / slots	2 pole pairs · 24 slots
Axial length · airgap	50 mm · 1 mm
Magnet	Br = 1.2 T, surface mounted
Angular discretisation	360 positions · 2-D A_z FEM
Cost	about 2 min per run

Why two routes. Maxwell stress and co-energy use different arithmetic on the same field. If they agree, the field is resolved enough for torque; if they diverge, the cause is usually the mesh or the angular discretisation — and the number should not ship.

POINT	MAXWELL (NM)	CO-ENERGY (NM)	DIFFERENCE
γ = −30°	14.7285	14.7452	0.11%
γ = 0° (design)	16.6967	16.7113	0.087%
γ = +30°	14.2502	14.2537	0.024%

At the design point the two routes sit **0.087%** apart — far below anything a design decision can resolve.

At low-torque points the relative disagreement grows: at cogging instants it reaches **2.6%** (0.3391 against 0.3302 Nm). Not a contradiction: the same absolute difference weighs more on a small number, which is why the check must also be read in absolute terms.

> **House rule.** Two independent routes do not prove the result is right — they prove it is consistent. The proof against reality is still a bench test, and this report has none.

QUANTITY	VALUE	NOTE
Cogging peak-to-peak	0.7447 Nm	period of 15° (360/24 slots)
Mean torque at 1,500 rpm	16.05 Nm	operating point evaluated
Magnet flux (peak)	0.1871 Wb	per phase
Back-EMF at 1,500 rpm	51.66 V peak · 40.36 V RMS	sinusoidal
Stator iron loss	9.70 W	at 50 Hz
Rotor iron loss	0.021 W	essentially zero
Magnet eddy currents	0.172 W	at 50 Hz

Cogging of 0.74 Nm peak-to-peak against 16.7 Nm rated is 4.5% ripple — the number that decides whether the machine needs skew or a fractional slot pitch.

On the same formulation, a topology optimization of the reluctance rotor: 1,410 design variables over 5,640 elements, gradient checked by finite differences with a maximum relative error of $\mathbf{4.6 \cdot 10^{-8}}$.

RESULT	VALUE
Solid rotor (reference)	≈ 0 Nm ($-2.4 \cdot 10^{-11}$)
Optimized rotor, linear model	38.10 Nm mean torque
Optimized rotor, nonlinear model	13.44 Nm mean torque
Co-energy torque at 5°	47.07 Nm
Iterations · time	120 · 215 s
Volume fraction	0.500

The difference that changes the decision. The same design delivers **38.1 Nm** in the linear model and **13.4 Nm** with saturation — a factor of 2.8. The linear number is useful for exploring the design space; it is not a machine number.

ITEM	WHAT THE REPORT SAYS
Dimensions and materials	illustrative; this is not a commercial machine
2-D FEM	no end effects, skew or eccentricity
Saturation	only in the nonlinear model; the linear one overshoots torque by 2.8×
Thermal	out of scope: losses are computed, not coupled
Cogging	from one set of rotor positions, not a fine sweep
Validation	no bench test; the check is between two numerical routes

Why publish a case without a test. Because internal consistency is verifiable and useful: a design that fails the Maxwell × co-energy check has a problem before it reaches the bench. What this report does not do is call consistency validation.

0.087%
between the two torque routes

4.5%
of cogging ripple

38.1 Nm
optimized rotor (linear)

13.4 Nm
same rotor, with saturation

What can be decided with this study

Comparing rotor topologies, choosing the current point and sizing the magnet, with the consistency check in place. To buy or certify a machine, a bench test and thermal are still missing.

Provenance

WHAT	WHERE
Full result	`PINNeAPPle/examples/motor_magnetics/results/spm_summary.json`
Script	`examples/motor_magnetics/01_spm_cogging_and_torque.py`
Rotor optimization	`examples/topology_optimization/results/reluctance_rotor_summary.json`

This is an example report, on demonstration data. A study for your machine follows the same structure: two torque routes, the difference between them published, and the page of what was not validated.