PINNeAPPle

STUDY REPORT · PUBLIC EXAMPLE

Topology optimization projection, gradient and the grey that stays

MBB and cantilever: four runs, a finite-difference gradient check and the effect of pushing the grey elements out.

−18.6%	$1.0 \cdot 10^{-6}$	49 s
of compliance with projection	maximum error in the gradient check	for the five optimizations

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Demonstration case: 2-D MBB beam and cantilever, SIMP
Finite-difference gradient check on every run
PINNeAPPle · LAIS & YAN LTDA · pinneapple-labs.github.io

Topology optimization answers a design question: **where to put material** so the structure is stiffer at the same mass. The answer is a density field, and what decides whether it is usable is the quality of that field — not the pretty picture.

CASE	WHAT IT IS
MBB beam	classic benchmark load case, maximum volume 50%
Cantilever	cantilever beam, maximum volume 40%
Formulation	SIMP with density filter and, in one run, projection
Optimizers	OC (optimality criteria) and MMA (method of moving asymptotes)

What is measured. Compliance (lower is better), final volume fraction, grey element fraction and the gradient check error. The number that matters is the combination — low compliance with little grey.

Every run ends with three checks: the volume is the one requested, the grey fraction is measured, and the analytical gradient is compared against finite differences.

CHECK	CRITERION	RESULT
Final volume	equal to the requested limit	0.500 · 0.500 · 0.500 · 0.400
Gradient	maximum relative error by finite differences	$1.0 \cdot 10^{-6}$ `passes`
Cost	total time of the five runs	49.4 s

The gradient check looks like paperwork until the day a result improves 20% with no explanation: with it, either the gradient is right and the gain is real, or the optimizer is walking down the wrong slope — and that shows up before it becomes a drawing.

RUN	COMPLIANCE	ITERATIONS	VOLUME	GREY FRACTION
MBB · OC, filter	233.71	146	0.500	0.444
MBB · MMA, filter	233.49	58	0.500	0.453
MBB · MMA, filter + projection	190.05	207	0.500	0.0225
Cantilever · MMA, projection	75.43	228	0.400	0.00875

OC and MMA land in practically the same place with the filter (233.71 against 233.49), with MMA in 58 iterations against 146. Projection changes the result for real: compliance drops to **190.05**, over 207 iterations.

> **The correct reading.** It is not “projection is better”: projection pushes density towards 0 or 1, which changes the physics of the problem being solved. Comparing the two numbers without the grey fraction would be comparing two different structures.

Without projection, **44%** of the elements sit at intermediate density — a material that does not exist. With projection that fraction falls to **2.25%** on the MBB and **0.88%** on the cantilever.

METRIC	MBB WITH FILTER	MBB WITH PROJECTION	EFFECT
Compliance	233.49	190.05	−18.6%
Grey fraction	0.453	0.0225	−95%
Iterations	58	207	×3.6 cost

The 18.6% compliance gain arrives with 3.6 times more iterations. The design decision is that trade — not an obvious choice.

ITEM	WHAT THE REPORT SAYS
2-D, linear elastic	no contact, buckling, plasticity or fatigue
Residual grey	2.25% on the MBB: the design is not fully 0/1 yet
Compliance	depends on filter, mesh and projection; numbers for one case, not a family
Manufacturability	not assessed: the result is not a production drawing
Cost	49 s for the set, on one machine — not a robustness sweep

From field to part there is one more step. Between the density field and the part there is geometric interpretation, tolerances and structural verification of the final geometry. This study delivers the material distribution; the part is another project.

190.05
compliance with projection

2.25%
grey elements

58
iterations with MMA + filter

$1.0 \cdot 10^{-6}$
error in the gradient check

What can be decided

Where to put material, at what computational cost, and whether the result is trustworthy (gradient checked, volume respected, grey measured). What does not come out of here is a manufacturable part.

Provenance

WHAT	WHERE
Results	`PINNeAPPle/examples/topology_optimization/results/mbb_summary.json`
Scripts	`examples/topology_optimization/01_mbb_beam.py`
Reluctance rotor	`results/reluctance_rotor_summary.json`

This is an example report, on public benchmarks. A study for your structure follows the same structure: volume respected, gradient checked, grey measured — and the page of what was not verified.